Moduli spaces in higher gauge theory

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Resumo: From the point of view of geometry, higher gauge theory (HGT) is a generalization of the notion of principal G-bundles with connection over a manifold M. At the first higher level, the Lie group G is replaced by a (strict) Lie 2-group $\mathcal{G}$. For the purposes of this talk we will view a Lie 2-group simply as a Lie crossed module, i.e. a pair of Lie groups (G, E), related by some properties. Taking a connection on a principal G-bundle to be a suitable collection of locally defined $L(G)$-valued 1-forms, where $L(G)$ denotes the Lie algebra of G, the corresponding notion of connection in HGT based on a 2-group is a suitable collection of locally defined $L(G)$-valued 1-forms and $L(E)$-valued 2-forms [1]. I will also comment briefly on the next level up, HGT based on a 3-group [2].

The main focus of the talk will be on how the familiar notion of moduli space of (the set of) flat G-connections on M modulo (the group of) gauge transformations generalizes to HGT based on a 2-group. I will discuss how a group action on a set generalizes to a 2-group action on a category [3], and use this to describe the higher moduli space in terms of the action of a 2-group of gauge transformations on a category of connections. Even for manifolds M of dimension 1 and 2, this already leads to interesting structure [4].

I conclude by presenting the case of finite groups and 2-groups, when M is a 2-manifold endowed with a cell structure. In this case there are counting invariants associated to the moduli spaces, leading to various group and 2-group identities [5].

palavras-chave: Higher gauge theory; 2-groups; moduli spaces.

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