Topology of moduli spaces of free group representations in reductive groups

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Resumo: Let G be a reductive algebraic group and Γ be a finitely generated group. Moduli spaces of representations of Γ into G, the so-called G-character varieties of Γ, play important roles in hyperbolic geometry, the theory of bundles and connections, knot theory and quantum field theories.

Let K be a maximal compact subgroup of G, and let F_r be a rank r free group. We show that the space of closed orbits in $\mathrm{Hom}(F_r, G)/G$ admits a strong deformation retraction to the orbit space $\mathrm{Hom}(F_r, K)/K$. In particular, all such spaces have the same homotopy type. We compute the Poincaré polynomials of these spaces for some low rank groups G. We also compare, for real G, the real moduli spaces to the real points of the corresponding complex moduli spaces, and describe the geometry of many examples.

palavras-chave: Character varieties; Reductive groups; Representation.

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