Regularity theory for time dependent mean field games

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Abstract We consider time dependent mean-field games (MFG) with a local power-like dependence on the measure and Hamiltonians satisfying both sub and superquadratic growth conditions. We establish existence of smooth solutions under a certain set of conditions depending both on the growth of the Hamiltonian as well as on the dimension. In the subquadratic case this is done by combining a Gagliardo-Nirenberg type of argument with a new class of polynomial estimates for solutions of the Fokker-Planck equation in terms of L^rL^P-norms of D_pH. These techniques do not apply to the superquadratic case. In this setting we recur to a delicate argument that combines the non-linear adjoint method with polynomial estimates for solutions of the Fokker-Planck equation in terms of $L^\infty L^\infty$-norms of D_pH. Concerning the subquadratic case, we substantially improve and extend the results previously obtained. Furthermore, to the best of our knowledge, the superquadratic case has not been addressed in the literature yet. In fact, it is likely that our estimates may also add to the current understanding of Hamilton-Jacobi equations with superquadratic Hamiltonians. Recent developments and perspectives conclude the talk. This is based on series of joint works with D. Gomes and H. Sanchez-Morgado.

keywords: Mean-field games, Hamilton-Jacobi equations, Fokker-Planck equations, classical solutions, a priori estimates